General Certificate of Education
June 2008
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MFP3**
Unit Further Pure 3

Monday 16 June 2008 1.30 pm to 3.00 pm

For this paper you must have:
- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = \ln(x + y)$$

and $$y(2) = 3$$

Use the improved Euler formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(2.1)$, giving your answer to four decimal places. *(6 marks)*

2 (a) Find the values of the constants a, b, c and d for which $a + bx + c\sin x + d\cos x$ is a particular integral of the differential equation

$$\frac{dy}{dx} - 3y = 10\sin x - 3x$$ *(4 marks)*

(b) Hence find the general solution of this differential equation. *(3 marks)*

3 (a) Show that $x^2 = 1 - 2y$ can be written in the form $x^2 + y^2 = (1 - y)^2$. *(1 mark)*

(b) A curve has cartesian equation $x^2 = 1 - 2y$.

Find its polar equation in the form $r = f(\theta)$, given that $r > 0$. *(5 marks)*

4 (a) A differential equation is given by

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$$

Show that the substitution

$$u = \frac{\mathrm{d}y}{\mathrm{d}x}$$

transforms this differential equation into

$$\frac{\mathrm{d}u}{\mathrm{d}x} - \frac{1}{x}u = 3x$$

(2 marks)

(b) By using an integrating factor, find the general solution of

$$\frac{\mathrm{d}u}{\mathrm{d}x} - \frac{1}{x}u = 3x$$

giving your answer in the form $u = \mathrm{f}(x)$. *(6 marks)*

(c) Hence find the general solution of the differential equation

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$$

giving your answer in the form $y = \mathrm{g}(x)$. *(2 marks)*

5 (a) Find $\int x^3 \ln x \, \mathrm{d}x$. *(3 marks)*

(b) Explain why $\int_0^{\mathrm{e}} x^3 \ln x \, \mathrm{d}x$ is an improper integral. *(1 mark)*

(c) Evaluate $\int_0^{\mathrm{e}} x^3 \ln x \, \mathrm{d}x$, showing the limiting process used. *(3 marks)*

6 (a) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 10\mathrm{e}^{-2x} - 9$$

(10 marks)

(b) Hence express y in terms of x, given that $y = 7$ when $x = 0$ and that $\frac{\mathrm{d}y}{\mathrm{d}x} \to 0$ as $x \to \infty$. *(4 marks)*

Turn over for the next question

7 (a) Write down the expansion of $\sin 2x$ in ascending powers of x up to and including the term in x^3. *(1 mark)*

(b) (i) Given that $y = \sqrt{3 + \mathrm{e}^x}$, find the values of $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $x = 0$. *(5 marks)*

(ii) Using Maclaurin's theorem, show that, for small values of x,

$$\sqrt{3 + \mathrm{e}^x} \approx 2 + \tfrac{1}{4}x + \tfrac{7}{64}x^2$$ *(2 marks)*

(c) Find

$$\lim_{x \to 0} \left[\frac{\sqrt{3 + \mathrm{e}^x} - 2}{\sin 2x} \right]$$ *(3 marks)*

8 The polar equation of a curve C is

$$r = 5 + 2\cos\theta, \qquad -\pi \leqslant \theta \leqslant \pi$$

(a) Verify that the points A and B, with **polar coordinates** $(7, 0)$ and $(3, \pi)$ respectively, lie on the curve C. *(2 marks)*

(b) Sketch the curve C. *(2 marks)*

(c) Find the area of the region bounded by the curve C. *(6 marks)*

(d) The point P is the point on the curve C for which $\theta = \alpha$, where $0 < \alpha \leqslant \frac{\pi}{2}$. The point Q lies on the curve such that POQ is a straight line, where the point O is the pole. Find, in terms of α, the area of triangle OQB. *(4 marks)*

END OF QUESTIONS